ATLS: TaR Proposal

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(Dated: November 3, 2020)

I. INTRODUCTION

Student understanding of the pre-requisite mathematics for physics and chemistry classes can largely impact both their academic achievement and self-confidence in these courses. [1, 2] In particular, upper level undergraduate students taking quantum mechanics must heavily draw on concepts from their previous courses in linear algebra, calculus, and statistics, then navigate how to apply these concepts to new physical principles that often go directly against their previously constructed scientific intuition. [3] Many instructors aim to help their students cultivate both a deep conceptual understanding of introductory quantum mechanics and a comfortable fluency with the underlying mathematical formalism. [4] But these two goals often compete for instructional time. Sometimes instructors choose to re-teach concepts from pre-requisite classes (or teach mathematical formalism not previously covered in math classes) during class to ensure student success, and interviews suggest that this leads to student perceptions that these classes strongly emphasize calculation at the sacrifice of robust conceptual frameworks. [5] On the other hand, quantum mechanics is notoriously difficult conceptually: numerous studies have shown that the physical misconceptions about the subject persist no matter whether it is taught at the secondary, [6] undergraduate, [7] and graduate [8, 9] levels. Thus, spending too much time reviewing and practicing mathematical formalism can prevent instructors from allocating adequate time for students to grapple with difficult concepts during face-to-face sessions.

The intense cognitive effort that students must undertake to grasp fundamental quantum mechanical concepts further highlights the importance of a solid mathematical background. In the context of a physics classroom, the pre-requisite mathematical manipulations and algorithms can be viewed as a foundation that require the use of "lower-order" Bloom's taxonomy skills, while physics concepts and applications can be understood as building on the foundation and using "higher-order" Bloom's taxonomy skills. Recent work looking into how fact-based and higher-order retrieval practice influences fact-based and higher-order test performance suggests that having a mix of retrieval practice questions increases performance on both fact-based and higher-order test questions, while practicing with only one level of retrieval questions only improves performance on test questions of the same level. [10] In typical introductory quantum mechanics courses, open-response questions on exams either implicitly or explicitly involve *both* lower-order facility with mathematical formalism and higher-order analysis of novel quantum systems. Thus, it is important for any students with weaker mathematical backgrounds to have access to and practice both fact-based mathematical review questions and higher-order quantum mechanics questions throughout the course.

A variety of curricular and pedagogical approaches have been attempted to ensure students in quantum mechanics understand the pre-requisite mathematical knowledge. Individual institutions have reported putting together an upper-level pre-requisite math sequence tailored to the needs of physical chemistry students [11] or revamping the introductory calculus sequence to better align with quantum chemistry learning objectives. [12] Other researchers have called for a complete rewrite of introductory mathematical sequences for chemistry students, [13] perhaps even mirroring integrated mathematics/physics courses that seem to be effective for introductory physics. [14] Similarly, in other educational contexts, summer bridge programs have been used to increase self-efficacy and shore up mathematical understanding of incoming college freshmen STEM majors [15] and to help "level the playing field" between public policy students with disparate quantitative backgrounds [16].

Recognizing that such sweeping institutional changes might not always be possible, this study focuses on supplemental mathematical instruction during introductory quantum mechanics, as this is more likely to be within an individual instructor's control. In addition to a curricular reworking, one of the aforementioned studies also adopted a test-bank of problems that students worked through before class to increase the percentage of chemistry majors that passed quantum chemistry. [12] Similar to that approach, this study proposes to provide students with guided mathematical resources that they can review before the corresponding concepts appear in physical contexts within the quantum mechanics class. However, my study constructs these *asynchronous* math review mini-modules as supplementary to the course's main learning objectives and emphasizes their optional nature, allowing students to self-evaluate their own mathematical knowledge and engage with them accordingly. Overall low student completion rates of massive open online courses (MOOCs), another asynchronous learning environment, suggest that these math mini-modules might go underutilized since they are not tied to a specific short-term goal, e.g. a homework grade. [17]

In short, this study proposes to examine correlations between students' prior mathematical knowledge, the effort they put towards curated online math review resources, and their subsequent understanding of the fundamental principles of quantum mechanics. After a brief pre-test and in-class explanation at the semester's start about the asynchronous review mini-modules, students will be given agency over how often and to what degree they engage with the lesson-aligned resources (and seek instructor help in specified math review online forum). By tracking how many review problems each stu-

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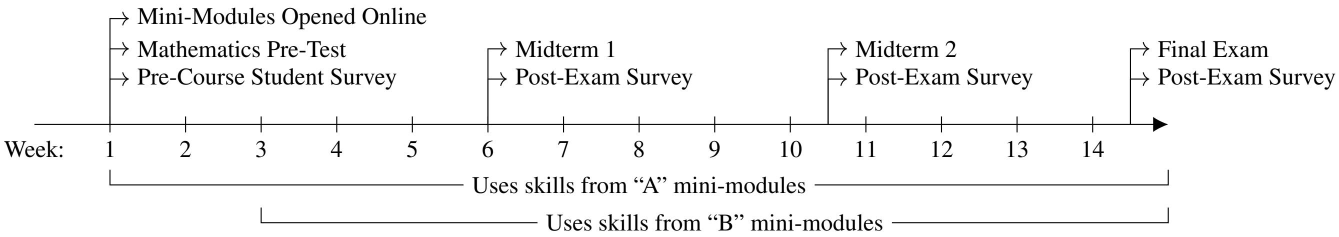

FIG. 1. Timeline of the intervention (asynchronous mini-modules) and data collection (exams, surveys) of this study

dent attempts over time, their performance on targeted exam questions, and their responses to post-exam surveys, I plan to investigate the following questions:

In an introductory quantum mechanics course, to what extent do undergraduates choose to engage with optional, targeted, asynchronous math review mini-modules? How does a student's level of participation affect their mastery of the corresponding physics-focused learning objectives, as measured by summative content assessments?

See Figure 1 for a schematic timeline of this proposed study.

II. METHODS

A. Study Population

I propose to carry out this study within the Physics Department's "Introductory Quantum Mechanics" fall semester class (14 calendar weeks and a finals period) at Knottareel University, a public 4-year research university. This course typically attracts third-year and fourth-year undergraduate students majoring in Chemistry, Phyiscs, and Chemical Physics and has an average enrollment of 70 students. The main instructional format of the class historically consists of two weekly mandatory, synchronous, in-person sessions ("Lecture") led by the instructor of record and a weekly, optional problem-solving discussion section ("Recitation") led by a graduate teaching assistant. Listed pre-requisites for the course include at least two semesters of introductory calculus-based physics, and at least one semester each of multivariate calculus and linear algebra. The course catalog also emphasizes that introductory statistics and two semesters of general chemistry are highly recommended; while these are not strict pre-requisites, many students have taken them or used AP/IB exam scores to gain college credit in these subjects. Students in physics/chemistry also tend to have taken an additional 1-4 courses in their respective departments before this class. Many chemistry students, particularly those more interested in biological rather than physical aspects of chemistry, opt to instead take the equivalent of Quantum Mechanics in the Chemistry Department.

B. Pedagogical Approach

After reviewing the syllabus during the first lecture of the semester, the instructor will explain to students that optional mini-modules will be available (on a companion website with anonymous login) to help review concepts from previous courses that will be necessary to learn quantum mechanics. The instructor will be sure to emphasize that surveys, tracking of engagement with the online platform, and forum posts/messages sent through the companion website will be anonymous so as to not bias the assignment of grades, with the instructor only being able to see student identities once the grade submission deadline for the semester has passed. Students will then be given approximately 15 minutes to complete the Mathematics Pre-Test in Appendix A and then immediately asked to complete the Pre-Study Survey given in Appendix B. After this initial data is collected, the instructor will continue to explain the general goals of the TaR project, to understand how these provided math mini-modules are used and see their effect on learning in the classroom. Students will be told that the mini-modules specifically will cover mathematical material that they are assumed to have seen in previous courses; thus these modules will provide opportunities for practice of this material before it is encountered in a physical context during the Lecture/Recitations, allowing for more of a focus on the difficult conceptual material and interesting applications of quantum mechanics during in-person sessions. In explaining the format of the asynchronous mathematical modules, the instructor will emphasize further resources to assist students in reviewing mathematical concepts and in particular the willingness of instructional staff to help students with mathematical concepts, even if they are not covered in synchronous sessions, to ensure their success in the class overall. Students will then be e-mailed a unique, anonymous ID they can use to log in to the companion math mini-module site.

A total of four mini-modules will be made before the semester starts, two for statistics and two for linear algebra. See Table I for a breakdown of the specific algorithms within these subjects that the mini-modules will cover. Though quantum mechanics also requires the extensive use of calculus, I chose not to focus on it in this study as students will already have had extensive practice applying calculus to physical problems in other physics classes. In contrast, many students may not have seen the corresponding concepts from linear algebra and statistics applied to a physical problem before. Within these broad subject areas, the specific topics outlined

Mini-Module	Learning Objectives
PDFs A	– Normalize a discrete probability distribution function (PDF) – Find the average and standard deviation of a function of a random variable given its discrete PDF – Find the probability of a random variable being within a set of values given its discrete PDF
PDFs B	– Normalize a continuous probability distribution function (PDF) – Find the average and standard deviation of a function of a random variable given its continuous PDF – Find the probability of a random variable being within a set of values given its continuous PDF
Eigenbasis A	– Determine whether a vector is an eigenvector of a matrix (and its corresponding eigenvalue if so) – Compute the eigenvalues and orthonormal eigenvectors of matrices by hand and by using mathematical software – Decompose a given vector in terms of the orthonormal eigenbasis of a matrix
Eigenbasis B	– Determine whether a function is an eigenfunction of an operator (and its corresponding eigenvalue if so) – Normalize a function over a specified domain – Find the overlap of two functions over a specified domain

TABLE I. Algorithmic learning objectives focused on by each mini-module

here were chosen because the corresponding physical applications of them in quantum mechanics, shown in Table II, are essential for grasping core learning objectives of the course. As an example, determining the probability of a particle in the infinite square well of being found within a certain region of position space requires students to (1) associate the square of the wavefunction with the probability distribution function for the particle's position and (2) integrate the probability distribution function over the specified region of space to find the probability. While the first step here is taught explicitly in the course, the second step is a purely mathematical algorithm that students are often assumed to have previously seen by the time they arrive in the quantum mechanics classroom. In Table I, the second step of the example here falls under the third learning objective that will be covered under the Mini Module "PDFs B".

Crucially, these mini-modules are not meant to be a comprehensive mathematical course in themselves. Instead, their emphasis is on practical fluency with the mathematical techniques, as the above example shows. Therefore, we will design the mini-modules to focus on algorithmic or process-oriented applications of the underlying mathematics (e.g., *Find the eigenvalues of the following Hermitian matrices.*), rather than abstract mathematical theory (e.g., *Give an example of a matrix A such that the algebraic multiplicity of one of the eigenvalues of A differs from its geometric multiplicity.*). Additionally, mathematical language used in these modules will be aligned to the greatest extent possible with language used in the standard textbook of the course, the instructor's lectures, and assessments to further help bridge the gap between students' previous mathematical understanding and their application of this knowledge to the context of quantum mechanics.

Within a mini-module, each learning objective is expected to take students between 30-90 minutes to complete all problems (dependent on individual background knowledge), though I expect that many students will not work through all problems nor engage with every learning objective. Each learning objective will have four parts: (1) introduction of the key terms, mathematical ideas, and mathematical algorithm to be practiced; (2) several guided, annotated practice problems with full explanations; (3) two multiple choice questions applying the algorithm with full explanations; (4) a set of practice problems without full explanations. Both multiple choice and open-response questions will be scored within the companion website's software, but students will have unlimited attempts and these scores will not factor into their course grades. For the multiple choice questions, an incorrect response will prompt the software to explain the error in reasoning that would lead one to select the response, while the correct choice will give a fully worked explanation. Students will have access to all explanations and errors in reasoning once they have selected the correct response. Though explanations for the open response questions will not be given, a "Show Answer" button will appear after 2 incorrect attempts at a question, allowing all students access to the same material whether they are successful in completing a question or not.

The instructor will explain to students that the mathematical algorithms contained in the mini-modules will be used in throughout the class, with mini-modules "A" being used from the beginning of the course and mini-modules "B" being used starting in Week 3 of the course (see Figure 1 for a full timeline). Students may choose to complete mini-modules in any order and can jump around to any learning objective that they wish using the companion website's table of contents. In addition to seeking help on mathematical concepts being encouraged during in-person office hours, the companion website will feature an anonymous discussion board and messaging system (instructor names will not be anonymous) where students can ask fellow classmates or the instructors for clarification. Though the anonymity may depersonalize students' social experience engaging with this material, it will also allow curtail embarrassment of asking about "pre-requisite" knowledge in front of instructors or fellow classmates, and seeing instructors actively respond to their concerns without judgment may help reduce student anxiety to approaching course staff in person as well.

Mini-Module Subject	Sample Applications in Quantum Mechanics
Probability Distribution Functions	– Normalization of the wavefunction/state vector – Expectation values and uncertainty of observables – Probability of system's momentum/energy/position etc. being measured within a range of values
Eigenbasis	– Possible values of a measurement of a system's momentum/energy/position/etc. – Determination of Hamiltonian eigenstates, whose time evolution is trivial – Interpretation of the square of the wavefunction as a probability density – Transition probabilities between quantum mechanical states

TABLE II. Correspondence of mini-module mathematical subjects to underlying quantum mechanical principles

C. Study Design

Our overall approach is quantitative, with a quasi-experimental design, since students will determine how often they themselves engage with the asynchronous online mini-modules. The Mathematical Pre-Test (Appendix A) consists of three multiple choice/multiple response questions from each of the two mini-module subjects, **Probability Distribution Functions** and **Eigenbasis**. Our analysis will categorize mathematical knowledge applied on assessments into these two categories, allowing us to test only half (6/12) of the mathematical review learning objectives. The Pre-Course Survey (Appendix B), given immediately after this pre-test, asks students their previous mathematical coursework and how confident they are in their knowledge of previously learned mathematical concepts. This is intentionally given after students take the pre-test and right before they are told about what the math mini-modules will review, in order to give an accurate picture the student self-assessment of their abilities at the time they are told about the supplemental review available.

Three exams, two midterms and a final, will be given throughout the semester at the weeks shown in Figure 1. Each student's exam score will be recorded as a measure of overall performance in the course. In addition, student responses to specific parts of scaffolded, multi-step open response questions will be dissected to determine when students correctly apply mathematical principles targeted by the mini-modules and their corresponding physical applications. 4-6 of these question parts will be targeted on each midterm, and 8-10 parts of questions will be targeted on the final exam. Note that all mathematical manipulations on summative assessments will have a physical component, i.e. none of these questions will directly mimic questions asked in the math mini-modules.

In the class directly following each exam, students will be asked to fill out the Post-Exam Questionnaire (Appendix C) that asks about the extent of their engagement with the math review mini-modules and other mathematical review resources they may have used to help themselves succeed in Quantum Mechanics. Questions about student use of the mini-modules will both serve to check measurements of engagement via problem completion in the mini modules as well as determine other measures that students use to review their mathematical knowledge. Open response questions at the end of the survey about student motivation for using or not using the mini-modules will be qualitatively coded. If the responses are overall too short/concise to give meaningful insight into student perceptions, follow-up interviews with 3-5 students may be conducted at the end of the semester to supplement information gathered in these surveys.

D. Data Collection

How does a student's level of participation affect their mastery of the corresponding physics-focused learning objectives, as measured by summative content assessments?

Learning analytics from the companion website will provide most of the quantitative data about *the extent to which students choose to engage with the asynchronous math review modules*. Specifically, the companion website will track whether and when students visit each topic within each mini-module, how many questions they attempt (and how many attempts they make before choosing the correct answer or asking to "Show Answer"), and when students post in the online forum. To help ensure consistency between the data gathered by the website and students' perceptions and experience, the Post-Exam Survey will also ask students to estimate how many problems in each topic they completed for that portion of the semester. Free response answers on the Post-Exam Survey (and follow-up interviews with a select number of students if necessary) will further help to contextualize the different ways in which students engaged with the mini modules and the extent to which they believe they benefited from this interaction.

I plan to measure whether *student participation in the mini-modules is correlated with mastery of physics-focused learning objectives* using both overall exam scores and coding of student responses on targeted exam questions. For each exam question part that is selected, researchers will code whether a student's response contains the correct physical reasoning/set-up and separately whether the mathematical algorithm coming from the mini-modules was performed correctly. Examples of this coding are shown in Table III for the example from the end of Section II B. Another confounding factor that may influence both student performance and engagement with the mini-modules is the extent of their previous knowledge, previous mathematical courses taken, and their confidence in the math skills they have previously learned. To account for this, student performance on the Mathematical Pre-Test and responses to the Pre-Course Survey will be recorded to account for these differences.

Question: What is the probability that a particle whose normalized wavefunction is $\Psi(x) = \sqrt{\frac{5}{32}}x^2$ will be found in the region $0 \leq x \leq 1$?

Sample Student Answer	Correct Math?	Correct Physics?
Probability = $\Psi(x)$, so $\sqrt{\frac{5}{32}}x^2 = \sqrt{\frac{5}{32}}(1)^2 \approx 39.53\%$	No	No
Probability Density Function = $\Psi(x)$, so $\int_0^1 \sqrt{\frac{5}{32}}x^2 dx = \sqrt{\frac{5}{32}}\frac{(x)^3}{3}\Big\|_0^1 \approx 13.18\%$	Yes	No
Probability Density Function = $\|\Psi(x)\|^2 = \frac{5}{32}x^4$, so $\frac{5}{32}x^4 = \frac{5}{32}(1)^4 \approx 15.63\%$	No	Yes
Probability Density Function = $\|\Psi(x)\|^2 = \frac{5}{32}x^4$, so $\int_0^1 \frac{5}{32}x^4 dx = \frac{5}{32}\frac{(x)^5}{5}\Big\|_0^1 \approx 3.13\%$	Yes	Yes

TABLE III. Sample student responses and their coding, as discussed in Section II D for a question corresponding to the mathematical/physical learning objective pair introduced in Section II B. Students must both square the wavefunction (physical learning objective) and integrate over the specified region (mathematical algorithm) in order to calculate the correct probability.

E. Limitations

One particular limitation of this study is that I may not have correctly determined which mathematical concepts students would most benefit from reviewing, and students may not engage with the mini-modules if they are too obvious/trivial. In this case, the conclusions that I would be able to draw from this study might be limited and anecdotal based on low participation. On the other hand, given that statistics is not a strict pre-requisite, students may become overwhelmed if they view the mini-modules collectively as an additional course in and of itself. To address this concern, I will work to ensure that the average content covered in class and pace will not be significantly changed from other years in which the course has been offered. In other words, learning objective expectations will not differ significantly from previous offerings of the course, so time students put into the mini-modules is expected to further solidify their physical understanding of the material. If all students are spending a large amount of time on these review mini-modules, then I may discover that many of the concepts that are assumed to be review based on pre-requisite courses may not be review at all, which would require further investigation.

Students may get the misconception at the beginning of the class that the mini-modules are mandatory or that they will get extra points for doing these mini-modules, even though their participation will be kept anonymous to the instructor until final grades have been submitted. By making sure to add clear language to the syllabus and careful wording on both the companion website and the Post-Exam Questionnaire, I hope to curtail any misgivings in this direction.

Students may also be afraid to use the companion site, comment on the companion site's forum with mathematical questions, or admit their engagement with these resources in the surveys due to feeling embarrassed about not knowing "pre-requisite" mathematical algorithms. Maintaining anonymity throughout engagement with the companion site and posting norms for civil discussion there will help to ensure that students feel more comfortable getting help from the instructional staff and their peers so that they can ultimately be better prepared to use these mathematical tools.

Finally, student abilities and application of the differing mathematical processes may vary within each of the two mini-module subjects, **Probability Distribution Functions** and **Eigenbasis**. Though I will not be able to make fully disentangle student performance on specific mathematical/physical learning objectives, my goals here are instead to examine student understanding of these two mathematical focus areas and its general correlations with performance on physically relevant learning objectives. With this preliminary data, further studies could work to elucidate greater nuance between student mathematical understanding and gains in physical intuition in upper-level physics courses.

III. ANTICIPATED RESULTS

To answer our first research question on student engagement, I plan to first document trends in student engagement over time with the four mini-modules. Aggregate page traffic on each of the four mini-modules, as well as the number of questions students answer in each mini-module, will be binned in approximately one-week intervals and plotted on a bar graph to show engagement over time. I expect to see several trends within this data: (1) a large contingent of students that work on the "A" modules or all modules before the first midterm to ensure they are prepared for the course; (2) a contingent of students that complete some amount of practice problems right before each of the three exams to study or cram; and (3) a smaller contingent of students that complete some amount of practice problems immediately after each of the exams as a head-start on improving their score for the next exam. I expect that overall there will be a difference of completion between the four mini-modules based on student self-confidence in the varying subjects. Qualitative data from the Post-Exam Questionnaire will help to clarify additional motivations students have for engaging with the mini-modules.

After general trends of student engagement are understood, I plan on examining correlations between the extent of each student's engagement with the two math subjects and three possible factors: their prior knowledge for each subject as measured by Mathematics Pre-Test scores, their confidence in their pre-existing knowledge of and self-confidence in Linear Algebra/Statistics as aggregated from their responses to the Pre-Course Student Survey, and the total grade on their previous summative assessment. I expect that overall, students are more likely to engage and attempt more practice problems in the mini-modules if they receive a lower score on the pre-test, report lower self-confidence and pre-existing knowledge on

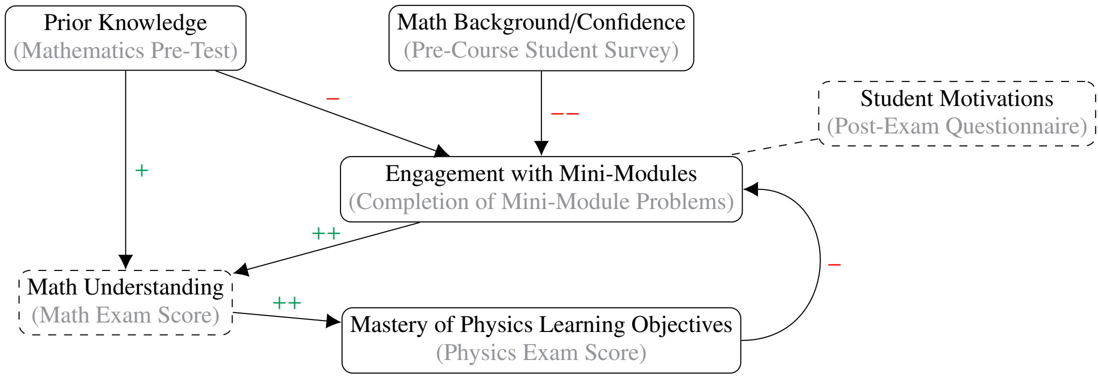

FIG. 2. Expected correlations between various quantities (and their measurements) in this study. A '+' indicates an expected positive correlation and a '−' indicates an expected negative correlation. The largest correlations expected are indicated with two symbols instead of one. The dashed line around *Math Understanding* indicates that it is a suspected intermediate mechanism for overall mastery of physics-based learning objectives.

the pre-course survey, or perform below the class average on the previous exam. I predict that student confidence/perceived background knowledge will be the strongest of these three effects, in particular when looking at student participation in these mini-modules in the first six weeks of the course (before the first midterm). If the amount of student participation is quite varied among the class but not strongly correlated with any of the above effects, this analysis would indicate that student motivation to complete optional, asynchronous math review modules in quantum mechanics is not largely influenced by their prior mathematical knowledge or performance.

To answer our second research question about the effect of engagement on student performance, I will first calculate exam math and physics scores for each students by counting the number of times each student correctly applies the mathematical algorithm or the physical principle to the selected exam question parts using the coding detailed in Section II D. For each exam and each of the two mathematical subjects, I will also sum (1) how many questions each student had attempted within that subject's mini-modules from the beginning of the course to the week of the exam and (2) the score of that student on the Mathematical Pre-Test questions pertaining to that subject. My main prediction is that the number of questions a student completes in a subject prior to an exam correlate positively with not only their math exam scores but also with their *physics exam scores*. Furthermore, I expect this correlation to be stronger for students with lower scores on the Mathematical Pre-Test. **Together, these predictions would suggest that my intervention of mathematical mini-modules may help close gaps in achievement in Introductory Quantum Mechanics between students with weaker and stronger mathematical backgrounds.** See Figure 3 for a graphical summary.

If there is no correlation found between the extent to which students who engage with the mathematical mini-modules and their mastery of the corresponding essential physical principles, this study will suggest that either (1) increasing mathematical fluency may not sufficiently help students grasp the underlying physical principles or (2) self-guided review of prior mathematical knowledge may not provide adequate preparation for applying it to a physical context. Both would be interesting findings to research in a follow-up study.

IV. DISCUSSION

Even after two years taking similar courses at a single university, undergraduates might enter an Introductory Quantum Mechanics course with a wide variety of mathematical backgrounds and fluencies. The surprising results of quantum mechanics often motivate curious students to take the course, but internalizing even the basic principles of quantum mechanics requires the upper levels of Bloom's taxonomy as they cannot rely on their previous physical or chemical intuition to help them analyze quantum systems. Through this project, I hope to show that providing students with guided mathematical review outside of class removes barriers to understanding of the most common algorithms used in Quantum Mechanics for students with less background knowledge in Statistics and Linear Algebra. At the same time, the passive instruction inherent in the asynchronous, ungraded nature of this intervention will hopefully allow for more time in face-to-face sessions to clarify difficult physical concepts using a mathematical language common to all students in the class. Finally, analyzing trends in student engagement with the mini-modules and reactions to their utility will provide me with a rough gauge of whether asynchronous review is an appropriate alternative to using in-class time to go over pre-requisite mathematical material. In any case, this project will allow me to glimpse at the rich interplay of student prior knowledge from previous STEM courses with the learning objectives of an upper-level physics course that contribute to their training as physical scientists over their undergraduate career.

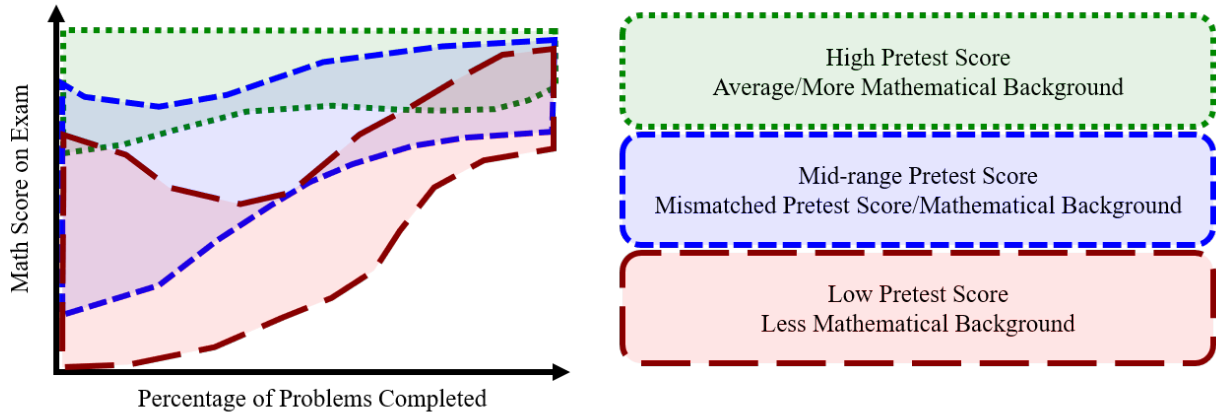

FIG. 3. Overall anticipated trends in results; shaded/outlined areas represent cohorts of students on the expected scatter plot. Students who complete more math practice problems from the optional review modules are expected to have better mathematical (and physics) knowledge on exams. The effect on student score is expected to be greatest for students with a lower pretest score, who will hopefully score higher on both aspects of exam questions if they complete more math practice problems. Mathematical background is self reported using the survey in Appendix B. A mismatched pretest score and mathematical background indicates students that report lots of mathematical background but do not score well on the pretest or vice versa, causing large variation within the middle cohort. Students who do not engage with the optional practice problems will likely also have large variation in their exam scores. The plot shown is with respect to math exam scores; I expect a similar trend for the physics exam scores, but with a lower average score for all students who do not complete any mathematical practice problems (the left end of the graph) and with less spread overall.

V. REFLECTION

The impetus for this TaR proposal was largely based in my own experience as a chemistry and physics undergraduate and my experience teaching high school physics. Drawing from when I felt frustrated as a student for being hindered by "hidden" learning objectives involving the application of prior knowledge in my upper-level physics classes, I made sure to do explicit review of algebraic material and to disentangle as much as possible a student's prior math knowledge from their physics comprehension so that all students had the chance to succeed in my class. Writing this proposal was particularly exciting as it gave me new ways to grapple with the question of how best to remove mathematical barriers based on prior knowledge from the undergraduate physics classroom. In devising a research question, I was forced to succinctly articulate the impact that I want to have on students (better mastery of physics-based learning objectives) and choose one possible avenue out of many as a pedagogical approach to work towards that goal (asynchronous math modules). In working through the details of how my intervention would be implemented, I had the space to fully explore many ideas that had never quite made it out of my head and onto paper. Some aspects of the proposal required tough decisions that helped me hone in on exactly what I thought would make the most impact: picking only a few mathematical concepts out of the myriad that students may have trouble with in upper level physics, balancing the length and number of surveys to give enough information yet not invoke survey fatigue, etc.

In between the end of ATLS and when I completed this proposal, I finished writing my first full scientific paper and wrote an original research proposal for a PhD requirement (separate from my doctoral research topic). Writing substantial research documents in both chemical and educational contexts has furthered my development as a researcher greatly. Specifically, seeing how the research skills that I am developing in a chemical context can be easily converted to an education context has given me a renewed sense of purpose and direction in grad school, since I do not plan on continuing computational chemistry research after getting my PhD. ATLS has been the first time in which I've done a literature search in the field of education/pedagogy, and finding evidence to back up my claims (and sometimes changing my claims based on literature evidence) within the introduction has increased my confidence in my own abilities to effectively search out educational research resources on my own. In the other direction, scaffolding the TaR proposal throughout ATLS allowed me to see the overall research/writing process from a new perspective and helped me craft the scientific manuscript that I wrote this summer.

Though I don't know my exact career trajectory, I am interested in teaching at community colleges as well as large, public universities with a teaching focus. Though I do not think that I will carry out this proposal exactly, I do want to implement the general idea of optional, asynchronous mathematical practice when I get back in the classroom, as I do believe it will help narrow gaps between students with disparate background mathematical knowledge. More broadly, I have recently become interested in the potential to study the transition from high school to post-secondary education and how new teaching practices and paradigms in both fields can inform each other. It was great to have the opportunity to put together my first TaR proposal here as practice, so that I can continue to propose and carry out many more TaR projects in the future as a professor.

Appendix A: Mathematics Pre-Test

All questions will be multiple choice/multiple response, and students will be given approximately 15-20 minutes to complete the pre-test. Students will be asked to leave a question blank only if they do not have an educated guess about the correct answer and reminded that these test scores do not count towards their grade. Draft questions are listed below, which will be piloted with a small group of graduate students (unrelated to the course) and edited to improve clarity.

1. Which of the following are eigenvalues or eigenvectors of the matrix $\begin{pmatrix} 2 & -i\sqrt{10} \\ i\sqrt{10} & -1 \end{pmatrix}$?
2. Suppose that one of the eigenvectors of a 3×3 matrix is found to be $\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$. Choose all of the following vectors that are correctly matched with their overlap with this eigenvector.
3. Which of the following functions are eigenfunctions of the operator $\frac{d^2}{dx^2}$?
4. Which of the following are normalized probability distribution functions over the range $0 \leq x \leq 10$, if x can only take integer values?
5. Which of the following gives the standard deviation of the random variable X with probability distribution function $p(x) = \frac{3}{8}x^2$ defined over the range $0 \leq x \leq 2$?
6. For which of the following (a, b) is $P(a \leq x \leq b) \geq 0.3$ for the random variable X with probability distribution function $p(x) = \frac{|x|}{4}$ defined over the range $-2 \leq x \leq 2$?

Appendix B: Pre-Course Student Survey

For Questions 1-4, please indicate whether you have taken the course in the past, are taking the course currently, or have not taken the course.

1. Single-variable Calculus
2. Multi-variable Calculus
3. Linear Algebra
4. Introductory Statistics (first semester)
5. Please list any other math courses you have taken in addition to those listed above since coming to college.

For Questions 6-9, indicate whether you Strongly Agree, Agree, Neither Agree nor Disagree, Disagree, or Strongly Disagree with the statement.

6. Regardless of the math courses I took in the past and my previous grades, I have a strong knowledge of single-variable calculus.
7. Regardless of the math courses I took in the past and my previous grades, I have a strong knowledge of multi-variable calculus.
8. Regardless of the math courses I took in the past and my previous grades, I have a strong knowledge of linear algebra.
9. Regardless of the math courses I took in the past and my previous grades, I have a strong knowledge of statistics/probability.
10. Is there any other information that you would like us to know to better understand your mathematical background coming into this course? Please use this space to explain.

Appendix C: Post-Exam Questionnaire

As mentioned at the beginning of the course, optional mini-modules on eigenvalues/vectors and probability distribution functions have been available for self-guided review. Thinking back over the time between the beginning of the course [after Midterm 1]/the last exam [after subsequent exams] and the exam you just took, . . .

1. . . . did you engage with the optional mini-modules?
   (a) I was not aware that the optional mini-modules existed.
   (b) I was aware of but did not engage with the optional mini-modules.
   (c) I engaged with the optional mini-modules in some way (reading, attempting problems, etc.)
2. . . . if you **did** engage, how did you engage with each of the two mini-module topics (Probability Distribution Functions A/B, Eigenbasis A/B)? Please check all that apply.
   (a) I did not engage with this topic.
   (b) I read over some or all of the information provided for this topic.
   (c) I attempted at least one problem in this topic.
   (d) I attempted at least ten problems in this topic.
   (e) I attempted at least thirty problems in this topic.
   (f) I attempted all (or almost all) of the problems in this topic.
   (g) I asked a question on the online forum or sent a direct message to an instructor about this topic.
   (h) I worked with a friend/tutor to review material in this topic.
   (i) I asked about this topic in office hours.
   (j) I reviewed notes from a previous class about this topic.

(k) I looked over an online source (such as LibreTexts) for help with this topic.

(l) I used another review strategy regarding this topic that isn't listed here. (please explain in the provided text box)

3. …why did you choose to use/not use the online mini-modules to the extent that you did?

4. …were there any other mathematical topics that you needed to spend significant time reviewing on your own during this portion of the class? If so, please describe these topics here.

5. …did your engage with the mini-modules more/less over this third of the course than the previous third? If so, why?

6. …how helpful did you find the existence of these mini-modules? Why did you find them helpful, not helpful, or unhelpful?

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